# Searching the Efficient Frontier for the Coherent Covering Location Problem

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***Abstract*****—**In this article, we will try to find an efficient boundary approximation for the bi-objective location problem with coherent coverage for two levels of hierarchy (CCLP). We present the mathematical formulation of the model used. Supported efficient solutions and unsupported efficient solutions are obtained by solving the bi-objective combinatorial problem through the weights method using a Lagrangean heuristic. Subsequently, the results are validated through the DEA analysis with the GEM index (Global efficiency measurement).

***Keywords*****—**Coherent covering location problem, efficient frontier, Lagrangian relaxation, data envelopment analysis.

## I. INTRODUCTION

A frequent problem in the public and private area is to determine the location of new facilities from where the points of demand are addressed. In order to support this decision, location problems are presented in the specialized literature. There are different ways to address this type of problems, like maximizing coverage or minimizing the total distance covered.

The problems that addressed the minimization of distance seeks that the total distance covered by the entire population to be as little as possible with a certain amount of facilities to install, while the problems that seeks to maximize coverage seek coverage for the population with a certain amount of installations. The localization problems with coverage includes the ”Set Covering Location Problems” (SCLP), which looks the minimum amount of facilities in order to cover the entire population and the ”Maximal Covering Location Problem” (MCLP), which looks for the maximum population covered given a limited amount of facilities to install (see [5], [6], [13] provides a review of the MCLP problems.

In literature, it is frequent to recognize the hierarchy models in the localization problems, which seek to determine the appropriated combination of service levels. In this type of models, the lower service levels provides a basic service, while a higher service level provides the most complete service in addition to the basic service, for example, if two types of facilities are considered (I and II) with two different services (A and B), the most basic service (A) shall be delivered by the type I facility (lower level) and also will be delivered by a type II facility (higher level), which in addition to the type A service, provides a type B service [19], [14] provides a review on the hierarchy localization problems).

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This work will look to find the efficient frontier for the Coherent Covering Location Problem presented by Serra [20], which proposes a bi-objective maximization model with two levels of hierarchy and a coherence relation.

This type of model is considered as a multiple-objective problem, where an important activity is to find the efficient frontier, which depends on the type of multiple-objective problem addressed: multiple-objective lineal problem or multiple-objective combinatorial problem. The methods development for the multiple-objective lineal problems cannot be directly applied to the multiple-objective combinatorial problems always, because the efficient frontier is formed by efficient solutions supported and efficient solutions not supported. Fig. 1 shows the different solution points of a bi-objective combinatorial problem, points A, B, C and D are efficient solutions supported that belongs to the convex hull, while points E and F are efficient solutions not supported. Points G, H and I are dominated solutions [23].

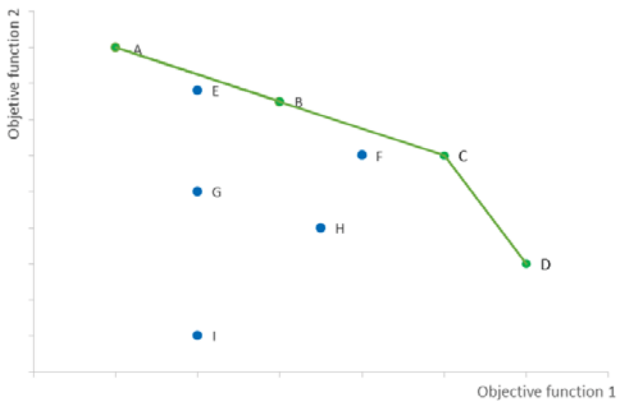


Fig. 1 Solutions point for a bi-objective combinatorial problem

The strategy to be used in order to obtain the efficient frontier for the localization problem with coherent coverage is through a bi-objective combinatorial problem in order to subsequently apply a lagrangian heuristic, where the efficient solutions supported, efficient solutions not supported and dominated solutions will be obtained. Subsequently, a data envelopment analysis will be applied in order to evaluate the efficiency of the solutions.

Section II presents the model to work, Section III explains the methodology to work in order to present the computer results in Section IV and finally, the conclusions are provided in Section V.

## II. Coherent Covering Location Problem

The Coherent Covering Location Problem (CCLP) is a two-level hierarchy problem that recognizes a relation of coherence between hierarchy levels. In this type of problem, the coherence is given when a population covered by a lower level installation must receive coverage by a single installation of the upper level. This model localizes two levels of facilities (I and II) where the coverage of the types of services (A and B) are maximized [20].

**Mathematical formulation**

Below is the mathematical formulation of the CCLP:

$i$ = Index of set of demand areas

$j$ = Index of potential sites for level I facilities

$k$ = Index of potential sites for level II facilities

$I$ = Set of demand areas

$J$ = Set of potential sites for facilities

$d_{ij}$ = DDistance between demand area $i$ and facility $j$

$S^{IA}$ = Threshold distance for level I facilities offering type A services

$S^{IB}$ = Threshold distance for level II facilities offering type A services

$T^{IB}$ = Threshold distance for level I facilities offering type B services

$S^{AB}$ = Maximun distance from a level I to a level II facility

$MA_i = \{j \in J \mid d_{ij} \leq S^{IA}\}$

$MB_i = \{k \in J \mid d_{ik} \leq S^{IB}\}$

$NB_i = \{k \in J \mid d_{ik} \leq T^{IB}\}$

$O_j = \{k \in J \mid d_{jk} \leq S^{AB}\}$

$h_i$ = Population at node $i$

$p$ = Facilities of level I

$q$ = Facilities of level II

**Variables**

$Z_i^A = 1$, if area $i$ is covered by a type A service; 0, otherwise

$Z_i^B = 1$, if area $i$ is covered by a type B service; 0, otherwise

$x_j = 1$, if there is a level I facility at $j$; 0 otherwise

$y_k = 1$, if there is a level II facility at $k$; 0 otherwise

**Objetive function**

$$Max \sum_{i \in I} h_i \cdot Z_i^A \tag{1}$$

$$Max \sum_{i \in I} h_i \cdot Z_i^B \tag{2}$$

**Restrictions**

$$Z_i^A \leq \sum_{j \in MA_i} x_j + \sum_{k \in MB_i} y_k \qquad \forall i \in I \tag{3}$$

$$Z_i^B \leq \sum_{k \in NB_i} y_k \qquad \forall i \in I \tag{4}$$

$$x_j \leq \sum_{k \in O_j} y_k \qquad \forall j \in J \tag{5}$$

$$\sum_{j \in J} x_j \leq p \tag{6}$$

$$\sum_{j \in J} y_j \leq q \tag{7}$$

$$x_j + y_j \leq 1 \qquad \forall j \in J \tag{8}$$

$$Z_i^A, Z_i^B, x_i, y_j \in 0,1 \quad \forall i \in I, j \in J \tag{9}$$

The objective function (1) looks to maximize the population that is covered by the type A service, while the objective function (2) looks to maximize the population covered by the type B service. The restrictions (3) indicates that a demand area $i$ is considered as covered by the type A service if there is a level I facility that is at a lower distance than $S^{IA}$ or a level II facility at a lower distance than $S^{IB}$. Even when the distances $S^{IA}$ and $S^{IB}$ refers to the same type of service, these would not be necessarily the same, since the facility of the level II for having more types of services shall be more attractive for population. The (4) restrictions establishes that a demand area $i$ is covered by the type B service if there is a facility of the level II at a lower distance than $T^{IB}$. The (5) restrictions are restrictions of coherence and establishes that a level I facility must be at a lower distance than $S^{AB}$ of a level II facility. Serra [20] performs an analysis on the values that must take the critical distance $S^{AB}$ in order to guarantee that there is coherence in the solution, there will be coherence if $S^{AB} \leq T^{IB} - S^{IA}$ (see Fig. 2). In the case that $S^{AB} > T^{IB} - S^{IA}$, coherence shall not be guaranteed (see Fig. 3). The (6) restriction establishes the maximum amount of level I facilities that can be located and the (7) restriction establishes the maximum amount of level II facilities to be located. The (8) restriction indicates that a level I and level II facility cannot be located in the same area and the (9) restrictions defines the nature of the decision variables.

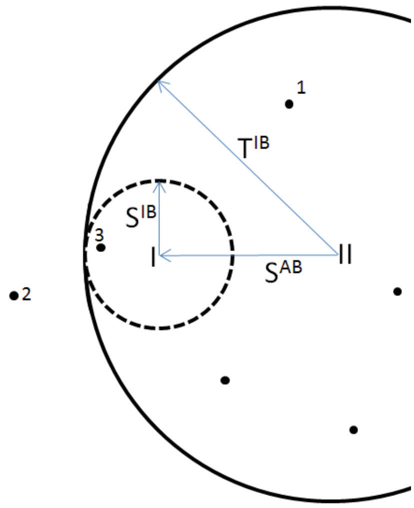


Fig. 2 Level I and II facilities offerins type A service with $S^{AB} \leq T^{IB} - S^{IA}$

## III. Efficient Border Construction

The CCLP is a bi-objective combinatorial model and in order to obtain an approximation of the efficient frontier, the

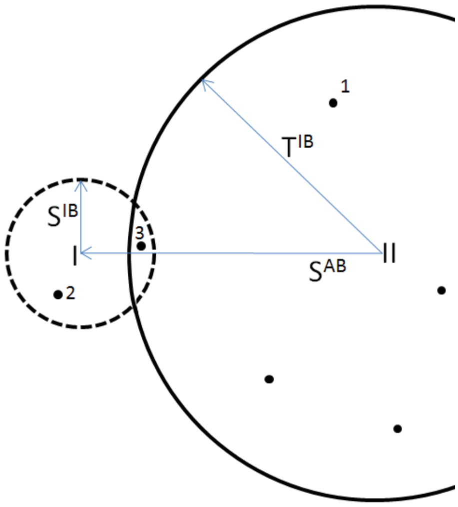


Fig. 3 Level I and II facilities offerins type A service with $S^{AB} > T^{IB} - S^{IA}$

weighting method based on the works of Alminyana et al. [1] shall be used in order to generate the efficient frontier of a problem of the bi-objective PQ-median and Espejo & Galvão [12] for a hierarchy localization problem of maximum bi-objective coverage.

In order to solve the problem, there would be three steps:

*Step 1:* Definition of the parametric problem for CCLP

Using the weight vector $(\alpha, (1-\alpha))$ the parametric problem for CCLP is defined CCLP:

$$CCLP(\alpha) = \ Max\ \alpha \cdot \sum_{i \in I} h_i \cdot Z_i^A + (1-\alpha) \cdot \sum_{i \in I} h_i \cdot Z_i^B \quad (10)$$

subject to (3)-(9).

*Step 2:* Solve the CCLP problem $(\alpha)$

In order to find feasible initial solution a vertexes replacement heuristic is used, localizing first the facilities of level II and subsequently level I.

Then, a lagrangian heuristic is performed for the resolution of the problem $CCLP(\alpha)$. In each iteration of the heuristic, a higher limit is obtained, from which it is possible to obtain a viable solution for the $CCLP$. The model results as follows:

$$v(CCLP(\alpha)_\lambda) = Max \quad \{\alpha \cdot \sum_{i \in I} h_i \cdot Z_i^A +$$

$$(1-\alpha) \cdot \sum_{i \in I} h_i \cdot Z_i^B + \sum_{j \in J} \lambda_j \cdot \left( x_j - \sum_{k \in O_j} y_k \right) \} \quad (11)$$

subject to 3 - 4, 6-9.

The part $\sum_{j \in J} \lambda_j \cdot \left( x_j - \sum_{k \in O_j} y_k \right)$ corresponds to the relaxation made, where $\lambda_j$ corresponds to the Lagrange's multiplier.

Next, based on the work of Galvão [17], the resolution logic is shown, including the update of the LaGranges multipliers made through the subgradient method:

$\lambda_i = 0$
$slack_i = 0$
$escala = 100$
$UB = \infty$
$LB = Initian\ solution$
**Begin While**
$Solve\ UB$
$If\ UB\ is\ equal\ in\ the\ 3\ last\ Iterations:$
$escala = \frac{escala}{2}$
$Obtaining\ LB\ based\ on\ the\ solution\ of\ UB$
$slack_j = x_j - \sum_{k \in O_j} y_k$
$norma = \sqrt{\sum_{j \in J} slack_j^2}$
$paso = escala \cdot \frac{UB - LB}{norma^2}$
$\lambda_j = max\,(0, \lambda_j + paso \cdot slack_j)$
$Save\ intermediate\ information$
$Verifications\ output\ condition$
**End While**

Two output condition verifications are performed: i) the difference between $UB$ and $LB$ must be lower than 1, this means, $(UB - LB < 1)$; ii) if after 500 iterations the last five values of $UB$ are equal, the cycle is finished and if there is any change, the cycle continues until 5 iterations have no change of $UB$.

*Step 3:* Validation of solutions

Fisher & Rushton [15] suggests to apply analytical techniques in order to validate the localizations proposed in the multiple-objective models. The Data Envelopment Analysis (DEA) is a frequent methodology (see [10]-[12], [1]) and it is used in order to validate the relative efficiency of the units, generally called DMUs (*Decision Making Units*) (see reference [9]).

There are two versions of DEA, depending on the hypothesis used for the construction of the DMUs set envelope: i) a version assuming a convex envelope (see for example the CCR models [4] or BCC [3]; ii) a version assuming a non-convex envelope (see for example the FDH models (*Free Disposable Hull*) of Tulkens [22].

A lagrangian heuristic generates a set of solutions that must be validated in order to determine the efficient solutions of the set. With this purpose and using the Relative Spatial Efficiency index (RSE).

In order to obtain the RSE of a solution, two types of efficiency are calculated: i) the technical efficiency given by the radial projection of the efficient frontier and II) the efficiency of the mix given by the values of the clearances of the DEA model used. Cooper & Tone [8] presents a discussion on the types of efficiency in DEA.

The RSE of a given solution consists in a combination of technical and mix efficiencies called Global Efficiency Measure (GEM): see Cooper & Pastor [7]. Since the CCLP efficient frontier consists in supported and not supported efficient solutions a non-convex envelope, the FDH methodology shall be used in order to determine the RSA of the solutions obtained with a lagrangian heuristic.

Obtaining the technical and mix efficiency shall be made through an adaptation of the Arnold's two-phase method [2. For each one of the solutions of the CCLP two steps are executed, step 1 where the technical efficiency is obtained and step 2 where the mix efficiency is obtained (maintaining the constant technical efficiency obtained in step 1).

Since that in the DEA models it is necessary to define*input* and *outputs* and the solutions obtained by the CCLP are a DMUs special case since it represents *outputs*, a non-convex adaptation of the model presented by Lovell & Pastor [18], it provides an economic point of view and a DEA model is accepted without *inputs*. The authors justify the use of models based on equivalences existing between the BCC models without *inputs* or BCC models with constant *inputs*.

With no loss of generality, it is assumed that the lagrangian heuristic found $t$ intermediate solutions for the CCLP. Where $\left(v_1^l, v_2^l\right)$ is the value of the solutions found in the iteration $l$, where $v_1^l$ and $v_2^l$ corresponds to the values of the objective function 1 and 2 respectively. The solution to be validated shall be denoted by $\left(v_1^0, v_2^0\right)$.

*First Part:* The technical efficiency of the solution to be validated is calculated by solving the following mixed integer programming model [18]:

$$Max \quad \{\phi_0\} \tag{12}$$

$$s.t. \quad \sum_{l=1}^{t} \lambda l \cdot v_r^l \geq \phi_0 \cdot v_r^0 \quad r = 1,2 \tag{13}$$

$$\sum_{l=1}^{t} \lambda_l = 1 \tag{14}$$

$$\lambda_l \in 0,1 \quad l = 1, ..., t \tag{15}$$

$$\phi_0 \; without \; restrictions \tag{16}$$

When $\lambda_l = 1$ it means that the solution $l$ is selected as reference in order to validate the solution $\left(v_1^0, v_2^0\right)$ ($\lambda_l = 0$ otherwise). In the objective function (12) a $\phi_0 = 1$ indicates that the validated solution has technical efficiency, while a $\phi_0 > 1$ indicates that the validated solution is technically inefficient. Equation (13) restriction prevents that the solution to be validated exceeds the efficient frontier. Equation (14) restriction indicates that a single solution must be selected as reference for the solution to be validated. Equations (15) and (16) restrictions defines the nature of the decision variables.

*Second Part:* Once the technical efficiency of the solution to be validated denoted by $\phi_0^*$ is obtained, the efficiency of the mix is calculated through the following mixed integer programming model [1]:

$$Max \quad \sum_{r=1}^{2} S_r \tag{17}$$

$$subject \; to \; (14), \; (15) \; and$$

$$\sum_{l=1}^{t} \lambda_l \cdot v_r^l - S_r = \phi_0^* \cdot v_r^0 \quad r = 1,2 \tag{18}$$

$$S_r \geq 0 \quad r = 1,2 \tag{19}$$

where $S_r$ is the clearance of each solution to validate. When the Objective Function (17) is equal than 0 it means that the validated solution has efficiency of the mix, otherwise the solution does not have efficiency of the mix.

After performing the two stages, the two types of efficiency are combined through the GEM indicator proposed by Cooper & Pastor [7]. It shall be denoted as $S_r^*$ the optimum value obtained in the second stage and the indicator to each solution to validate shall be applied

$$GEM = \frac{1}{\phi_0^* \left(1 + \frac{1}{2} \cdot \sum_{r=1}^{2} \frac{S_r^*}{\phi_0^* \cdot v_r^0}\right)} \tag{20}$$

where a $GEM = 1$ represents that the solution has efficiency regarding the other solutions, while a $GEM < 1$ indicates that the solution is inefficient.

## IV. Computer Results

The calculations were made in a computer with a 2.5 GHz Intel Core processor and 4 GB of RAM memory. The programming of Step 1 was made in *Python*, while the programming of Step 2 and 3 were in *AMPL* language with *CPLEX* as optimizer. In order to obtain the approximation of the efficient frontier for CCLP using a lagrangian heuristic, 41 instances were ran for each one of the problems. The following values of $\alpha \in [0, 1]$ were selected for the weighing method: $\alpha = \{(0; 1), (0, 025; 0, 975), (0, 050; 0, 950), ..., (1, 0)\}$.

In order to validate the results, it was decided to generate the exact efficient frontiers for the networks of 55 and 100 nodes; these were generated with the method of restrictions and programmed in *AMPL* language by using *CPLEX* as optimizer.

The computer results obtained with the methodology proposes is shown in the network of 55 nodes of Swain [21] (Table I) and for the network of 100 nodes of Galvão & ReVelle [16] (Table II). In addition, it shows graphs of the solutions obtained for the network of 55 nodes (Figs. 4 and 5) and for the network of 100 nodes (Figs. 6 and 7).

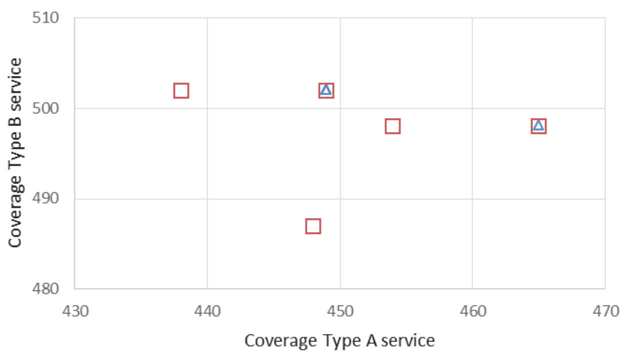


Fig. 4 Network of 55 nodes: Efficient frontier obtained by the method of the constraints (△) and solutions obtained by lagrangean heuristics (□). $n = 55,\ p = 1,\ q = 2\ S^{IA} = 6,\ S^{IB} = 8,\ T^{IB} = 10,\ S^{AB} = 4$

## V. Conclusion

In the article, a methodology was used in order to solve a bi-objective combinational problem, with coherence and two levels of hierarchy (*CCLP*).

TABLE I
RESULTS FOR A NETWORK OF 55 NODES OF SWAIN ($S^{IA} = 6,\ S^{IB} = 8,\ T^{IB} = 10,\ S^{AB} = 4$)

| Problem | | Method of the constraints | | Lagrangean heuristics | | | |
|---|---|---|---|---|---|---|---|
| $p$ | $q$ | Total time (segs. CPU) | Num. Sol. efficient | Total time (segs. CPU) | Num. Sol. | Num. Sol. con GEM=1 | Prom. de GEM |
| 1 | 1 | 0,4218 | 1 | 6.143,45 | 4 | 1 | 0,7606 |
| 2 | | 0,7187 | 2 | 8.371,55 | 14 | 1 | 0,7169 |
| 3 | | 0,7031 | 2 | 3.261,20 | 14 | 1 | 0,6478 |
| 1 | 2 | 0,8281 | 2 | 5.763,44 | 5 | 2 | 0,8594 |
| 2 | | 1,0000 | 3 | 6.462,00 | 19 | 2 | 0,7326 |
| 3 | | 1,1718 | 3 | 6.562,11 | 32 | 2 | 0,6628 |
| 4 | | 1,2968 | 4 | 5.025,86 | 30 | 2 | 0,6204 |
| 1 | 3 | 1,9531 | 4 | 5.669,23 | 8 | 2 | 0,8924 |
| 2 | | 2,8750 | 6 | 5.399,34 | 14 | 4 | 0,8343 |
| 3 | | 2,8125 | 6 | 4.914,81 | 34 | 3 | 0,7580 |
| 4 | | 3,2968 | 6 | 5.823,61 | 31 | 3 | 0,6529 |
| 5 | | 3,3437 | 6 | 4.964,05 | 33 | 3 | 0,6462 |
| 6 | | 3,4687 | 6 | 5.176,84 | 36 | 3 | 0,6297 |

TABLE II
RESULTS FOR A NETWORK OF 100 NODES OF GALVÃO & REVELLE ($S^{IA} = 40,\ S^{IB} = 50,\ T^{IB} = 60,\ S^{AB} = 20$)

| Problem | | Method of the constraints | | Lagrangean heuristics | | | |
|---|---|---|---|---|---|---|---|
| $p$ | $q$ | Total time (segs. CPU) | Num. Sol. efficient | Total time (segs. CPU) | Num. Sol. | Num. Sol. con GEM=1 | Prom. de GEM |
| 10 | 8 | 4,5156 | 4 | 29.192,0 | 25 | 4 | 0,8964 |
| 12 | | 4,5000 | 4 | 27.830,9 | 25 | 4 | 0,8959 |
| 14 | | 5,0156 | 4 | 23.820,6 | 27 | 4 | 0,8979 |

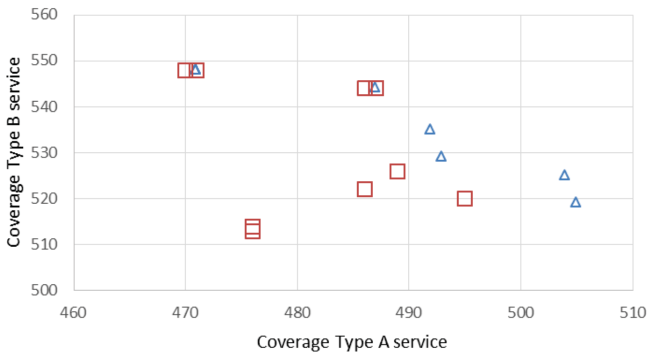


Fig. 5 Network of 55 nodes: Efficient frontier obtained by the method of the constraints (△) and solutions obtained by lagrangean heuristics (□). $n = 55,\ p = 2,\ q = 3\ S^{IA} = 6,\ S^{IB} = 8,\ T^{IB} = 10,\ S^{AB} = 4$

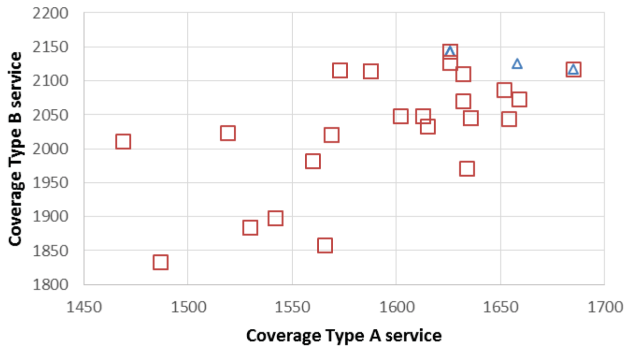


Fig. 7 Network of 100 nodes: Efficient frontier obtained by the method of the constraints (△) and solutions obtained by lagrangean heuristics (□). $n = 100,\ p = 10,\ q = 8\ S^{IA} = 40,\ S^{IB} = 50,\ T^{IB} = 60,\ S^{AB} = 20$

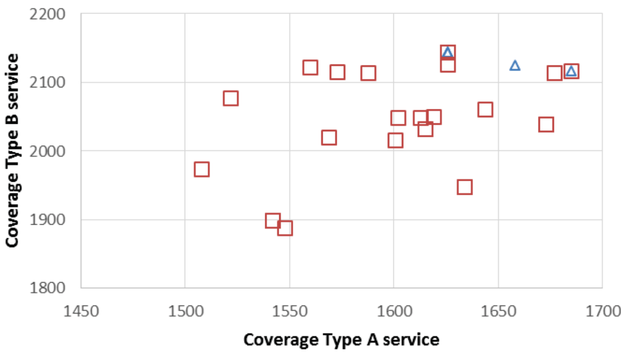


Fig. 6 Network of 100 nodes: Efficient frontier obtained by the method of the constraints (△) and solutions obtained by lagrangean heuristics (□). $n = 100,\ p = 12,\ q = 8\ S^{IA} = 40,\ S^{IB} = 50,\ T^{IB} = 60,\ S^{AB} = 20$

Solutions used during the process of search of the optimum solution of the lagrangian dual were used in order to find an approximation to the efficient frontier for the *CCLP*, where efficient solutions supported and efficient solutions not supported were obtained.

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